Example 1- Simplify
$$\frac{5x^2-15x}{10x^2}$$

Step 1- Make the expression a single fraction This step is already done.

Step 2- Completely factor the numerator (GCF, binomials/trinomials/special products)

$$5x^2 - 15x = 5x(x - 3)$$

Step 3- Completely factor the denominator (GCF, binomials/trinomials/special products)

This step is already done.

$$\frac{5x^2 - 15x}{10x^2} = \frac{5x(x-3)}{10x^2} = \left(\frac{x-3}{2x}\right)$$

Example 2- Simplify
$$\frac{m^2-m-2}{m^2+m}$$

Step 1- Make the expression a single fraction This step is already done.

Step 2- Completely factor the numerator (GCF, binomials/trinomials/special products)

$$m^2 - m - 2 = (m - 2)(m + 1)$$

Step 3- Completely factor the denominator (GCF, binomials/trinomials/special products)

$$m^2 + m = m(m+1)$$

$$\frac{m^2 - m - 2}{m^2 + m} = \frac{(m - 2)(m + 1)}{m(m + 1)} = \left(\frac{m - 2}{m}\right)$$

Example 3- Simplify
$$\frac{w^2-5w+6}{w^2-7w+12}$$

Step 1- Make the expression a single fraction This step is already done.

Step 2- Completely factor the numerator (GCF, binomials/trinomials/special products)

$$w^2 - 5w + 6 = (w - 3)(w - 2)$$

Step 3- Completely factor the denominator (GCF, binomials/trinomials/special products)

$$w^2 - 7w + 12 = (w - 3)(w - 4)$$

$$\frac{w^2 - 5w + 6}{w^2 - 7w + 12} = \frac{(w - 3)(w - 2)}{(w - 3)(w - 4)} = \left(\frac{w - 2}{w - 4}\right)$$

Example 4- Simplify $(r^2 - 5r + 4)(r - 4)^{-2}$

Step 1- Make the expression a single fraction

We will deal with the negative exponent to make a single fraction.

$$(r^2 - 5r + 4)(r - 4)^{-2} = \frac{r^2 - 5r + 4}{(r - 4)^2}$$

Step 2- Completely factor the numerator (GCF, binomials/trinomials/special products)

$$r^2 - 5r + 4 = (r - 4)(r - 1)$$

Step 3- Completely factor the denominator (GCF, binomials/trinomials/special products)

This step is already done.

$$\frac{r^2 - 5r + 4}{(r-4)^2} = \frac{(r-4)(r-1)}{(r-4)(r-4)} = \frac{r-1}{r-4}$$

Example 5- Simplify $\frac{x^2+2x-8}{(2-x)(4+x)}$

Step 1- Make the expression a single fraction This step is already done.

Step 2- Completely factor the numerator (GCF, binomials/trinomials/special products)

$$x^{2} + 2x - 8 = (x + 4)(x - 2)$$

Step 3- Completely factor the denominator (GCF, binomials/trinomials/special products)

This step is already done.

Step 4- Reduce common factors

$$\frac{x^2 + 2x - 8}{(2 - x)(4 + x)} = \frac{(x + 4)(x - 2)}{(2 - x)(4 + x)} = \frac{x - 2}{2 - x}$$

But we aren't done. We can factor -1 out of 2-x ... watch what happens! (2-x)=-1(-2+x)=-1(x-2)

Now we can rewrite the denominator

$$\frac{x-2}{2-x} = \frac{x-2}{-1(x-2)} = \frac{x-2}{-1(x-2)} = \frac{1}{-1} = \boxed{-1}$$